Ten Cool Calculus Tips

What follows are ten tips, warnings, strategies, etc. that can really come in handy in your calculus studies. Give 'em a try.

General

All calculus problems can be solved with one or more of the following approaches: *algebraic, graphical,* and *numerical.* When possible, use two or three of the approaches. Each approach gives you a different perspective on a problem and enhances your grasp of the relevant concepts.

Making commonsense estimates enhances mathematical understanding. When doing calculus, or any math for that matter, come up with a common sense, ballpark estimate of the solution to a problem before doing the math (when possible and time permitting). This deepens your understanding of the concepts involved and provides a check to the mathematical solution. This is a powerful math strategy.

Limits

The limit at a hole: The only way a function can have a regular, two-sided limit where it is not continuous is where the discontinuity is an infinitesimal hole in the function. And when you've got such a hole in a function, *the limit at the hole is the height of the hole.*

Ordinary math doesn't work with infinity (or zero to the zero power). When doing limit problems (or other calc problems), you can't use the ordinary rules of arithmetic when dealing with infinity (or zero to the zero power). For example, $\frac{\infty}{\infty}$ does not necessarily equal 1 (sometimes it does, but often it does not). By the same token, it might look like $\infty - \infty$ should equal zero, but it doesn't. And don't make the mistake of concluding that any of the following unequal things are equal: $0 \cdot \infty \neq 0, \frac{0}{0} \neq 1, 0^0 \neq 1, \infty^0 \neq 1,$ and $1^\infty \neq 1$.

Differentiation

Don't mix up the slopes (and derivatives) of horizontal and vertical lines. How steep is a flat, horizontal road? Not steep at all, of course. Zero steepness. So, a horizontal line has a slope of *zero* and, therefore, a derivative of zero. What's it like to drive up a vertical road? You can't do it. And you can't get the slope of a vertical line — it doesn't exist, or, as mathematicians say, it's *undefined*. So, naturally, if a function is vertical at any point or points, its derivative there is also undefined.

pi and *e* are numbers, not variables! And constants also behave like numbers. Don't forget that π (~3.14) and e (~2.72) are numbers, not variables, so they behave like ordinary numbers. Constants in problems, like c and k also behave like ordinary numbers. So, if you're doing a calculus problem that contains a pi or a c or k, etc. and you're not sure how to deal with it, imagine that the pi or c or k was, instead, an ordinary number like 5. However you would do that altered, simplified problem is precisely how you should do the original problem.

Psst, what's the derivative of cosecant? Imagine you're taking a test and you can't remember the derivative of tangent, cotangent, secant, or cosecant. You lean over to the guy next to you and whisper, "psst, what's the derivative of csc$x$?" Now, the last three letters of *psst* (sst) are the initial letters of sec, sec, tan. Write these three functions down, and below them write their cofunctions: csc, csc, cot. Put a negative sign on the csc in the middle. Finally, add arrows like in this diagram:

$$\begin{array}{ccccc} \text{sec} & \rightarrow & \text{sec} & \leftarrow & \text{tan} \\ \text{csc} & \rightarrow & -\text{csc} & \leftarrow & \text{cot} \end{array}$$

(This mnemonic device might seem complicated, but you'll remember the word *psst*, and after that the diagram is very easy to remember.)

Look at the top row. The *sec* on the left has an arrow pointing to *sec tan* — so the derivative of sec$x$ is sec$x$ tan$x$. The *tan* on the right has an arrow pointing to *sec sec,* so the derivative of tan$x$ is $\sec^2 x$. The bottom row works the same way except that both derivatives are negative.

With the chain rule, don't use two derivative rules at the same time. One way to reduce chain rule mistakes is to remember that you never use two derivative rules at the same time.

For example, when using the chain rule to differentiate $\ln(x^3)$, you first use the derivative rule for the outside function, the natural log function ($(\ln u)' = \frac{1}{u}$), then, as a *separate step,* you use the power rule to differentiate the inside function, x^3. The answer: $\frac{1}{x^3} \cdot 3x^2 = \frac{3}{x}$ At no point in any chain rule problem do you differentiate both the outside and the inside functions at the same time. For example, with $\ln(x^3)$, you do not use the natural log rule and the power rule at the same time to come up with. $\frac{1}{3x^2}$.

Three cases where the derivative does not exist.

- There's no tangent line and thus no derivative at any type of *discontinuity:* removable, infinite, or jump. Continuity is, therefore, a *necessary* condition for differentiability. It's not, however, a *sufficient* condition as the next two cases show. Dig that logician-speak.
- There's no tangent line and thus no derivative at a sharp *corner* on a function (or at a *cusp*, a really pointy, sharp turn).
- Where a function has a *vertical tangent line* (which occurs at a vertical inflection point), the slope is undefined and thus the derivative fails to exist.

Integration

The *LIATE* mnemonic. Herbert E. Kasube came up with the acronym *LIATE* to help you choose your *u* (you can check out Herb's article in the *American Mathematical Monthly* 90, 1983 issue):

L	Logarithmic	(like $\log(x)$)
I	Inverse trigonometric	(like $\arctan(x)$)
A	Algebraic	(like $5x^2 + 3$)
T	Trigonometric	(like $\cos(x)$)
E	Exponential	(like 10^x)

To pick your *u* chunk, go down this list in order; the first type of function on this list that appears in the integrand is the *u*. Once you've selected your *u*, everything else is automatically the *dv* chunk.