Practice Test 3: Answers and Explanations

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Vou've likely come to this chapter because you just completed Practice Test 3 in Chapter 17. I hope you fared well. To find out, spend some time with this chapter to check your answers. I provide detailed explanations along with a bare-bones answer key. The answer key is great for when you're checking your answers in a hurry, but I suggest that you review the detailed explanations for each question — even those you answered correctly. The explanations show you how to calculate each problem and often provide tips and tricks.

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Mathematics Test

- 1. **D.** Two candy bars and two bags of potato chips cost \$4.00, and when you add in one more candy bar, the cost goes up by \$0.75. So you know that a candy bar costs \$0.75. Thus, two candy bars cost \$1.50. Deduct this amount from \$4.00 to determine how much two bags of potato chips cost: \$4.00 \$1.50 = \$2.50. So one bag of potato chips costs \$1.25.
- 2. **G.** Start with the highest number in the answers and work your way down: The number 10 is a factor of 60 but not 64, so Choice (K) is wrong. The number 8 isn't a factor of 60, so Choice (J) is wrong. The number 6 is a factor of 60 but not 64, so Choice (H) is wrong. The number 4 is a factor of both 60 and 64, so the correct answer is Choice (G).
- 3. **D.** The 5 children each received 24 stones, so the total number was $5 \times 24 = 120$. If 6 children had been present, each would have received $120 \div 6 = 20$.
- 4. **H.** The perimeter of the square is 20, so each side is 5 (because $20 \div 4 = 5$). Use the area formula to find the area of the square:

 $A=s^2=5^2=25$

The shaded region is half the area of the square: $25 \div 2 = 12.5$.

- 5. **D.** Exactly 9 people ordered beef stroganoff (25% of 36 = 9), and 17 people ordered chicken divan. So 9 + 17 = 26 people *didn't* order linguini primavera. As a result, you know that 36 26 = 10 people ordered this pasta dish.
- 6. **K.** Begin by substituting –1 for *v* and 4 for *w*:

 $4v(w^2 - 3vw) = 4(-1)((4)^2 - 3(-1)(4))$

Evaluate using the standard order of operations:

= 4(-1)(16+12) = 4(-1)(28) = -112

7. A. If you let w equal the amount of time it took Annette to walk across the width of the field, then 4w equals the time she took to walk the length. Annette walked around the perimeter of the field in 10 minutes, so plug these values into the equation for the perimeter of a rectangle:

P = 2l + 2w10 minutes = 2(4w) + 2w

Simplify the right side of the equation:

10 minutes = 8w + 2w10 minutes = 10w

Divide both sides by 10:

1 minute = w

So Annette would take 1 minute to walk across the width of the field.

8. **G.** From left to right, count *down* 6 and *over* 3. Then change this statement into a fraction: $\frac{-6}{3}$. (*Down* means you need a negative and *over* indicates the division bar.)

Now reduce the fraction to get your answer:

$$-\frac{6}{3} = -2$$

9. D. Isolate the absolute value on one side of the equation:

|2v - 13| + 6 = 9|2v - 13| = 3

Now split the absolute value into two separate equations and solve both:

2v - 13 = 3	2v - 13 = -3
2v = 16	2v = 10
v = 8	<i>v</i> = 5

The value of 8 + 5 = 13.

10. **H.** Let *x* be Damien's score on the fourth day. So his four scores were 93, 92, 89, and *x*; his average was 90. Plug these numbers into the formula for the mean and simplify:

$$Mean = \frac{Sum \text{ of values}}{Number \text{ of values}}$$
$$90 = \frac{93 + 92 + 89 + x}{4}$$
$$90 = \frac{274 + x}{4}$$

Solve for *x*:

360 = 274 + x86 = x

11. B. Multiply both sides of the equation by 2 to eliminate the fraction, and then simplify:

$$7-3p \ge \frac{5}{2}$$
$$14-6p \ge 5$$
$$-6p \ge -9$$

Divide both sides by –6 and reverse the inequality, and then reduce the resulting fraction:

$$p \le \frac{-9}{-6}$$
$$p \le \frac{3}{2}$$

12. K. Line *M* and line *N* are parallel, so the following equivalencies are true:

 $\angle a = \angle e$ $\angle c = \angle f$

Thus, $\angle a + \angle b + \angle f = \angle a + \angle b + \angle c = 180^\circ$. So you know that Choice (F) is wrong. And $\angle a + \angle d = \angle e + \angle d = 180^\circ$, so Choices (G) and (J) are both wrong as well. The three angles in a triangle add up to 180° , so $\angle b + \angle e + \angle f = 180^\circ$, thus Choice (H) is wrong. By the process of elimination, the correct answer is Choice (K).

13. A. Plug the coordinates for the two points into the distance formula:

Distance =
$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(6 - (-3))^2 + (-5 - 7)^2}$$

Then simplify to get your answer:

$$=\sqrt{9^2 + (-12)^2} = \sqrt{81 + 144} = \sqrt{225} = 15$$

14. **F.** To begin, find the greatest common factors (GCF) of the coefficient, the *x*, and the *y* for the expression $4x^2y^4 - 12x^3y^2 - 8xy^3$:

The GCF of 4, 12, and 8 is 4. The GCF of x^2 , x^3 , and x is x.

The GCF of y^4 , y^2 , and y^3 is y^2 .

Thus, $2x^2$ isn't a factor of $4xy^2$.

15. **D.** Of the 126 students who applied for the scholarship, 9 received it and 117 didn't. You can set up and simplify the ratio like this:

$$\frac{9}{117} = \frac{1}{13}$$

16. **K.** A parallelogram is a quadrilateral, so its angles add up to 360°. Opposite angles are equal, so any two adjacent angles add up to 180°. Thus, you can set up the following equation:

$$z - 15 + 2z = 180$$

 $3z - 15 = 180$
 $3z = 195$
 $z = 65$

Therefore

 $\angle D = z^\circ - 15^\circ = 65^\circ - 15^\circ = 50^\circ$

Because opposite angles in a parallelogram are equal, $\angle D = \angle B$, so y = 50.

17. E. To begin, solve 2x - y = 32 for y in terms of x:

2x - y = 32-y = -2x + 32y = 2x - 32

Next, substitute 2x - 32 for y in 5x + 3y = 14 and solve for x:

5x + 3(2x - 32) = 14 5x + 6x - 96 = 14 11x = 110x = 10

Now substitute 10 for x in y = 2x - 32:

$$y = 2(10) - 32 = 20 - 32 = -12$$

Therefore, xy = (10)(-12) = -120.

18. **K.** Begin by getting a common denominator of 3*b* on the left side of the equation, and then add the two fractions and simplify:

$$\frac{a}{b} + \frac{a+2}{3b} = \frac{1}{4}$$
$$\frac{3a}{3b} + \frac{a+2}{3b} = \frac{1}{4}$$
$$\frac{3a+a+2}{3b} = \frac{1}{4}$$
$$\frac{4a+2}{3b} = \frac{1}{4}$$

Now cross-multiply to remove the fractions:

$$4(4a + 2) = 3b(1)$$

Simplify and isolate the *a* term:

$$16a + 8 = 3b$$
$$16a = 3b - 8$$

Finally, divide both sides by 16 to get your answer:

$$a = \frac{3b-8}{16}$$

19. A. Translate the statement "*p* percent of 250 is 75" into an equation and solve for *p*:

$$p(0.01)(250) = 75$$

 $2.5p = 75$
 $p = 30$

Thus, 75% of 30 = 22.5.

20. **F.** To find the slope of 9x - 3y = 10, put the equation in the slope-intercept form:

$$9x - 3y = 10$$
$$-3y = -9x + 10$$
$$y = 3x - \frac{10}{3}$$

21. **D.** To find the coordinates, simply use the midpoint formula:

Midpoint =
$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-6 + 9}{2}, \frac{1 + 8}{2}\right) = \left(\frac{3}{2}, \frac{9}{2}\right)$$

22. J. To begin, split |6 - 4n| > 1 into two equations and remove the absolute value bars:

$$6 - 4n > 1$$
 $6 - 4n < -1$

Solve the first equation:

$$6-4n > 1$$
$$-4n > -5$$
$$n < \frac{5}{4}$$

Then solve the second equation:

$$6 - 4n < -1$$
$$-4n < -7$$
$$n > \frac{7}{4}$$

Therefore, $n < \frac{5}{4}$ or $n > \frac{7}{4}$.

23. E. The sine equals the opposite side over the hypotenuse:

$$\sin\theta = \frac{O}{H} = \frac{2}{\sqrt{29}}$$

To remove the radical from the denominator, multiply both the numerator and the denominator by $\sqrt{29}$:

$$=\frac{2}{\sqrt{29}}\cdot\frac{\sqrt{29}}{\sqrt{29}}=\frac{2\sqrt{29}}{29}$$

24. G. First, use the formula for the surface area to find the radius:

$$A = 4\pi r^{2}$$
$$36\pi = 4\pi r^{2}$$
$$36 = 4r^{2}$$
$$9 = r^{2}$$
$$3 = r$$

Next, plug the radius into the formula for the volume:

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(3)^3 = \frac{4}{3}\pi(27) = 36\pi$$

25. A. First, isolate the radical on the left side:

$$\sqrt{2x+3} - 1 = x$$
$$\sqrt{2x+3} = x+1$$

Next, square both sides of the equation (be sure to square the whole side in each case):

$$(\sqrt{2x+3})^2 = (x+1)^2$$

 $2x+3 = x^2+2x+1$

Now simplify and solve for *x*: $3 = x^2 + 1$

$$3 = x^{2} + 2 = x^{2}$$
$$\pm \sqrt{2} = x$$

26. G. Begin by listing the factors of 75:

Factors of 75: 1	3	5	15	25	75
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Thus, *m* and *n* could be any of these six values. A bit of trial and error working with these numbers reveals that 3 + 15 = 18.

- 27. **B.** The four percentages shown in the graph add up to 23% + 21% + 22% + 25% = 91%. Thus, Zach owns the remaining 9%. When he divides this percent into four equal parts, each part will be $9\% \div 4 = 2.25\%$. When this amount is added to Bell's, her total shares will be 25% + 2.25% = 27.25%.
- 28. **K.** According to the chart, the first balloon costs \$4.00, so begin building your function with f(x) = 4. Each balloon after the first costs an additional \$0.50, so you need to subtract 1 from *x* (to account for the first balloon) and then multiply this by 0.50:

f(x) = 4 + 0.50(x - 1)

Simplify the function to get the correct answer:

f(x) = 4 + 0.50x - 0.50f(x) = 3.50 + 0.50xf(x) = 0.5x + 3.5

29. **A.** Solve $4x^2 - 3x - 1 = 0$ by factoring:

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4x^2 - 3x - 1 = 0(4x + 1)(x - 1) = 0
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Split this into two equations and solve for *x*:

4x + 1 = 0 x - 1 = 0 4x = -1 x = 1x = -0.25

Therefore, the sum of these two values is -0.25 + 1 = 0.75.

30. H. First, use matrix addition:

 $| 4 \ 5 | + | 7 \ -3 | = | 11 \ 2 |$

Now multiply the result by 2:

2 11 2 = 22 4

31. **D.** The equation for the line is $y = \frac{1}{3}x + 3$, so the *y*-intercept is 3; therefore, k = 3. To find the value of *n*, plug in *n* for *x* and 5 for *y* in the equation:

$$y = \frac{1}{3}x + 3$$
$$5 = \frac{1}{3}n + 3$$
$$2 = \frac{1}{3}n$$
$$6 = n$$

Therefore, n = 6, so $kn = 3 \times 6 = 18$.

32. **F.** Let *a* equal Alfred's number and *r* Rani's. Then, translate the statements in the question into two equations as follows:

$$5a + 2r = 300$$
$$2a + 3r = 252$$

Multiply the first equation by 3 and the second equation by 2, and then subtract the second from the first:

$$15a + 6r = 900$$

- 4a + 6r = 504
11a = 396

Solve for *a*:

a = 36

Plug in 36 for *a* in either equation (the second equation looks easier), and then solve for *r*:

$$2(36) + 3r = 252$$

72 + 3r = 252
3r = 180
r = 60

Therefore, the sum of the two numbers is 36 + 60 = 96.

33. **C.** To begin, put 2x - 4y = 13 into the slope-intercept form:

$$2x - 4y = 13$$
$$-4y = -2x + 13$$
$$y = \frac{1}{2}x - \frac{13}{4}$$

The slope of this line is $\frac{1}{2}$. Thus, the slope of the line you're looking for is the negative reciprocal of $\frac{1}{2}$, which is -2. This line passes through the origin, so its *y*-intercept is 0. Plug these two values into the slope-intercept form:

y = mx + by = -2x + 0y = -2x

34. **J.** Let *x* equal the score that Gerry received on each of his first two exams. Plug in his scores, the number of exams he took, and his average into the formula for the mean:

$$Mean = \frac{Sum \text{ of values}}{Number \text{ of values}}$$
$$93 = \frac{2x + 94 + 85 + 90}{5}$$

Solve for *x* by first multiplying both sides of the equation by 5 to eliminate the fraction:

465 = 2x + 269196 = 2x98 = x

35. **A.** To begin, multiply both sides by y + 5 to eliminate the fraction:

$$\frac{2x+3y-19}{y+5} = 3$$

2x+3y-19 = 3(y+5)

Distribute on the right side:

2x + 3y - 19 = 3y + 15

Now subtract 3*y* from both sides, and then solve for *x*:

2x - 19 = 152x = 34x = 17

36. H. From the first experiment to the second:

The value of *w* is multiplied by 3.

The value of *x* is divided by 2.

The value of *y* is multiplied by 2.

The value of z is multiplied by 3.

Thus, the only two possible conclusions would be that w is directly proportional to z and that x is inversely proportional to y.

- 37. **A.** The *amplitude* is the measure of a wave from the vertical center to the crest. Therefore, this wave has an amplitude of 0.5.
- 38. **K.** \overline{OD} and \overline{OE} are both radii of the circle, so they're the same length. \overline{AB} is tangent at D, so \overline{AB} and \overline{OD} are perpendicular to each other. Similarly, \overline{BC} is tangent at E, so \overline{BC} and \overline{OE} are perpendicular to each other. Thus, three angles of *DBEO* are right angles, so the fourth angle is also a right angle. And adjacent sides \overline{OD} and \overline{OE} are the same length, so *DBEO* is a square.
- 39. E. The transformation to move f(x) three units to the right is f(x 3). Then, to reflect this across the *x*-axis, change it to -f(x 3).
- 40. **G.** The parabola is concave down, so *a* is negative. It's shifted to the right, so *a* and *b* have different signs; therefore, *b* is positive. It crosses the *y*-axis above the origin, so *c* is positive. Therefore, *a* can't be greater than *b*.

41. **D.** The value inside the radical must be greater than or equal to 0. Thus, if x = 0, the value inside the radical is:

 $x^2 - 9 = 0^2 - 9 = -9$

This value is impossible, so you can rule out Choices (A), (B), and (C). Additionally, the value of the denominator can't equal 0. If x = 3, the value of the denominator is

$$\sqrt{x^2 - 9} = \sqrt{3^2 - 9} = \sqrt{9 - 9} = 0$$

This value is also impossible, so Choice (E) is ruled out as well, leaving Choice (D) as the correct answer.

- 42. J. The sequence includes all multiples of 3, including 57 and 72, so Choices (H) and (K) are ruled out. The sequence also includes every number that's 1 added to a multiple of 3, which includes 34 (33 + 1) and 43 (42 + 1), so Choices (F) and (G) are ruled out. By the process of elimination, Choice (J) is the correct answer.
- 43. **A.** Ninety numbers exist in the range from 10 to 99, and 18 of them are divisible by 5. Place these two numbers into the formula for probability:

Probability =
$$\frac{\text{Target outcomes}}{\text{Total outcomes}} = \frac{18}{90} = \frac{1}{5}$$

44. **F.** The base of the triangle is the distance from (-5, -3) to (-2, -3), so it equals 3 units. The height of the triangle extends from y = -3 to y = 4, so it measures out to be 7 units. Plug these values into the formula for the area of a triangle:

$$A = \frac{1}{2}bh = \frac{1}{2}(3)(7) = 10.5$$

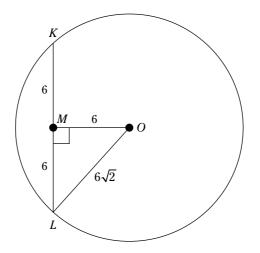
- 45. **B.** Every triangle has three angles that add up to 180° , and an isosceles triangle has two equivalent angles. Therefore, the three angles of this triangle are either $40^{\circ} + 40^{\circ} + 100^{\circ}$ or $40^{\circ} + 70^{\circ} + 70^{\circ}$. In any case, this triangle can't include a 50° angle.
- 46. **H.** To begin, FOIL 5 + 6i and 3 2i, and then combine like terms:

 $(5+6i)(3-2i) = 15 - 10i + 18i - 12i^2 = 15 + 8i - 12i^2$

Now substitute -1 for i^2 and combine like terms again:

= 15 + 8i - 12(-1) = 15 + 8i + 12 = 27 + 8i

47. **C.** $\overline{KL} \perp \overline{MO}$, and \overline{KL} is a chord of the circle, so \overline{MO} bisects \overline{KL} and KM = 6 and ML = 6. Thus, you can make a right triangle as follows:



This right triangle has two sides with the length of 6, so its hypotenuse is $6\sqrt{2}$. This value is also the radius of the circle, so plug it into the formula for the area of a circle:

$$A = \pi r^2 = \pi r^2 = \pi \left(6\sqrt{2} \right)^2 = 72\pi$$

- 48. J. Eldridge opened her account with \$1,000, so the line on the graph begins above the origin, which rules out Choices (G) and (H). The withdrawal caused the amount in the account to decrease, so the slope of the line decreases during this month. So Choice (F) is wrong. And in every month except the month with the withdrawal, the upward slope of the line is consistent, which rules out Choice (K). Through the process of elimination, you know that Choice (J) is the correct answer.
- 49. **C.** Eldridge started with \$1,000, made 8 deposits of \$200 each (8 × 200 = 1,600), and withdrew \$350, so 1,000 + 1,600 350 = 2,250.
- 50. **H.** Let *x* equal the original price for the shoes. The sale price was 0.7x, to which a 5% tax was added. So Martha paid 1.05(0.7x), and this amount was \$58.80. Thus, you set up the following equation:

1.05(0.7x) = 58.8

Simplify and solve for *x*:

0.735x = 58.8 $x = \frac{58.8}{0.735}$ x = 80

Therefore, Martha paid \$80 for the shoes.

51. **E.** To begin, use the identity $\tan n = \frac{\sin n}{\cos n}$ to substitute $\frac{\sin n}{\cos n}$ for $\tan n$:

 $\frac{\tan n \csc n}{\sin n \sec n} = \frac{\sin n \csc n}{\cos n \sin n \sec n}$

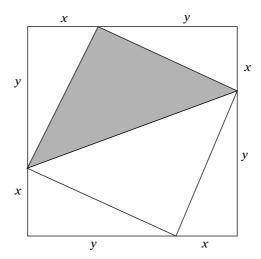
So you can cancel sin *n* in the numerator and denominator

$$=\frac{\csc n}{\cos n \sec n}$$

Now the identity $\cos n = \frac{1}{\sec n}$ means that $\cos n \sec n = 1$; therefore, the denominator is 1, so you can drop it:

$$=\frac{\csc n}{1}=\csc n$$

52. F. To get a sense of how to proceed, add a few lines to the given figure:



Notice that the shaded triangle is now half of an inner square. Each side of this square is the hypotenuse of a right triangle with legs of length *x* and *y*. Use the Pythagorean theorem to find the length of this hypotenuse:

$$a2 + b2 = c2$$
$$x2 + y2 = c2$$
$$\sqrt{x2 + y2} = c$$

Thus, the length of a side of this square is $\sqrt{x^2 + y^2}$, so its area is $x^2 + y^2$. The shaded triangle is half of this square, so its area is $\frac{x^2 + y^2}{2}$.

53. C. Multiply the two equations together:

$$\frac{f}{g} \cdot \frac{g}{h} = \frac{1}{4} \cdot \frac{2}{5}$$
$$\frac{f}{h} = \frac{2}{20}$$
$$\frac{f}{h} = \frac{1}{10}$$

54. **K.** Begin by squaring both sides of the equation to eliminate the radical. I do this in two steps to keep things clear:

$$49^{3y} = \sqrt{7^{y+1}}$$
$$(49^{3y})^2 = (\sqrt{7^{y+1}})^2$$
$$49^{6y} = 7^{y+1}$$

Now substitute 7² for 49 on the left side. Again, I do this in two steps:

$$\left(7^{2}\right)^{6y} = 7^{y+1}$$
$$7^{12y} = 7^{y+1}$$

Because the bases are both 7, the exponents are equal. So now you can drop the bases:

12y = y + 1

Solve for *y*:

$$11y = 1$$
$$y = \frac{1}{11}$$

55. **B.** Let *x* equal the area of the base of the pyramid and *y* equal the height of the pyramid. Plug these values into the formula for the volume of a pyramid:

$$V = \frac{1}{3}A_bh = \frac{1}{3}xy$$

So *x* equals the area of the base of the cylinder and 2*y* equals the height of the cylinder. Now plug these values into the formula for the area of a cylinder, substituting *x* for πr^2 :

$$V = \pi r^2 h = x(2y) = 2xy$$

Make a fraction using the area of the pyramid as the numerator and the area of the cylinder as the denominator, and then cancel out *xy* in the numerator and denominator:

$$\frac{\frac{1}{3}xy}{2xy} = \frac{\frac{1}{3}}{2}$$

Multiply the numerator and the denominator by 3 to eliminate the extra fraction:

- $\frac{1}{6}$
- 56. **K.** Multiply both sides of the equation by x + 2 to remove the fraction, and then place all terms on one side of the equation:

$$x = \frac{2k}{x+2}$$
$$x^{2} + 2x = 2k$$
$$x^{2} + 2x - 2k = 0$$

The result is a standard quadratic equation, where a = 1, b = 2, and c = -2k. Use the quadratic formula to solve:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{2^2 - 4(1)(-2k)}}{2(1)}$$

Now simplify:

$$=\frac{-2\pm\sqrt{4+8k}}{2}=\frac{-2\pm2\sqrt{1+2k}}{2}=-1\pm\sqrt{1+2k}$$

Thus, either of the following two solutions is valid:

$$-1 + \sqrt{1 + 2k}$$
 $-1 - \sqrt{1 + 2k}$

So in the first solution, reversing the two terms gives you $\sqrt{1+2k} - 1$.

57. **D.** Begin by factoring on both sides of the equation:

$$\frac{a^2 + 2ab + b^2}{a^2 - b^2} = 2a + 2b$$
$$\frac{(a+b)(a+b)}{(a+b)(a-b)} = 2(a+b)$$

Cancel (a + b) in the numerator and denominator:

$$\frac{(a+b)}{(a-b)} = 2(a+b)$$

Next, cancel (a + b) on both sides of the equation:

$$\frac{1}{(a-b)} = 2$$

Multiply both sides by (a - b):

$$1 = 2(a - b)$$

Now divide both sides by 2:

$$\frac{1}{2} = a - b$$

- 58. **J.** The value of a linear function inside absolute value bars can never be less than 0. Thus, a function with a range of $f(x) \ge 4$ takes this minimum value and adds 4 to it. Therefore, f(x) = |x 4| + 4 can never be less than 4, so the correct answer is Choice (J).
- 59. C. To begin, put the logarithm in exponential form:

$$\log_a \sqrt{b} = \frac{1}{4}$$
 means $a^{\frac{1}{4}} = \sqrt{b}$

Square both sides to remove the radical on the right side:

$$\left(a^{\frac{1}{4}}\right)^2 = b$$
$$a^{\frac{1}{2}} = b$$

Now square both sides again to isolate *a*:

$$\left(a^{\frac{1}{2}}\right)^2 = b^2$$
$$a = b^2$$

60. **H.** The arc length from *A* to *B* is 6π . The angle from *A* to *B* is $\frac{3}{8}$ of the circle's total of 360°, which equals 135°. Plug these values into the formula for arc length:

Arc length = degrees $\left(\frac{\pi r}{180}\right)$ $6\pi = 135 \left(\frac{\pi r}{180}\right)$

Solve for the radius *r*:

$$6\pi = \frac{3\pi r}{4}$$
$$24\pi = 3\pi r$$
$$8 = r$$

Now plug in 8 for *r* in the formula for the area of a circle:

Area = $\pi r^2 = \pi (8)^2 = 64\pi$

The area of the shaded region is $\frac{3}{8}$ of this value:

$$64\pi \left(\frac{3}{8}\right) = 24\pi$$